AP Calculus AB Summer Assignment

As advanced placement students, your first assignment for the 2023-2024 school year is to come to class the very first day in top mathematical form.

Calculus is a “world of change”. While words like limit, continuity, derivative and integral may seem foreign and daunting to you, I assure you that at this time next year at this time they will seem as commonplace to you as “graduation” and senioritis”!

I have prepared four assignments to help you strengthen your mathematical muscles! Make sure all of your work is in a logical order while indicating your answers clearly. Write neatly and in pencil. I will not give credit to anything that is sloppy or has no supporting work. In addition, please do not hand in work on paper torn from spiral notebooks. Hole punched plain paper or graph paper is preferred.

This packet is due the periodically throughout the summer. Below is a list of due dates. Spend some quality time on this packet and get off to a good start! I will be grading the work as you hand it in. Two thirds (2/3) of the AP exam next year is taken without a calculator. Only use a calculator on problems that indicate calculator use.

Assignment dates are listed below. They must be in the main office by noon on those dates.

Summer Assignment Part 1: Due by noon July 6th
Summer Assignment Part 2: Due by noon July 27th
Summer Assignment Part 3: Due by noon August 10th
Summer Assignment Part 4: Due the first Day of Class
In addition, there will be a quiz on trigonometry values on the first day of school!

If you have any questions about any of these problems or techniques used in solving them, you may contact me at the school website/email address: bquick@barnegatschools.com

Have a great summer!

Below are listed topics in the review. You can certainly do Google searches for any of these topics. But we have given you several sites that will cover pretty much all of these topics.
Here is a good site for most algebra topics:
http://www.purplemath.com/modules/index.htm

Beginning algebra topics
Exponents
Negative and fractional exponents
Intermediate algebra topics
Domain
Solving inequalities: absolute value
Solving inequalities: quadratic
Special Factoring formulas
Function transformation
Factor theorem (*p* over *q* method)
Even and odd functions
Solving quadratic equations and quadratic formula

Advanced algebra topics
Asymptotes
Complex fractions
Composition of functions
Solving Rational (fractional) equations

Trig Information
http://www.mathematicshelpcentral.com/index.html

Once in the site, go to lecture notes.
Basic right angle trig
Trig equations

Summer Assignment Part 1: Due July 6, 2023 - NON CALCULATOR

Name: **Cell:** **email:**

Section A: Simplifying Algebraic Expressions

In pre-calculus, you will be required to perform algebraic manipulations with negative exponents as well as fractional exponents. By definition: $x^{-n} = \frac{1}{x^n}$ and $x^{a/b} = \sqrt[b]{x^a} = \left(\sqrt[b]{x}\right)^a$

As a reminder, rules of exponents are as follows:

When we multiply, we add exponents $x^a \cdot x^b = x^{a+b}$

When we divide, we subtract exponents $\frac{x^a}{x^b} = x^{a-b}$, $x \neq 0$

When we raise powers, we multiply $\left(x^a\right)^b = x^{a \cdot b}$

Simplify and write with positive exponents.

1. $-8x^{-2}$

$\frac{-8}{x^2}$

2. $\left(-5x^3\right)^{-2}$

$(-5)^{-2}x^{3-2} = \frac{1}{(-5)^2 x^6} = \frac{1}{25x^6}$

3. $\left(-\frac{3}{x^4}\right)^{-2}$

$\frac{(-3)^{-2}}{\left(x^4\right)^{-2}} = \frac{1}{(-3)^2 x^{-8}} = \frac{x^8}{9}$

4. $\left(36x^{10}\right)^{1/2}$

$\sqrt{36x^{10}} = 6x^5$

5. $\left(27x^3\right)^{-2/3}$

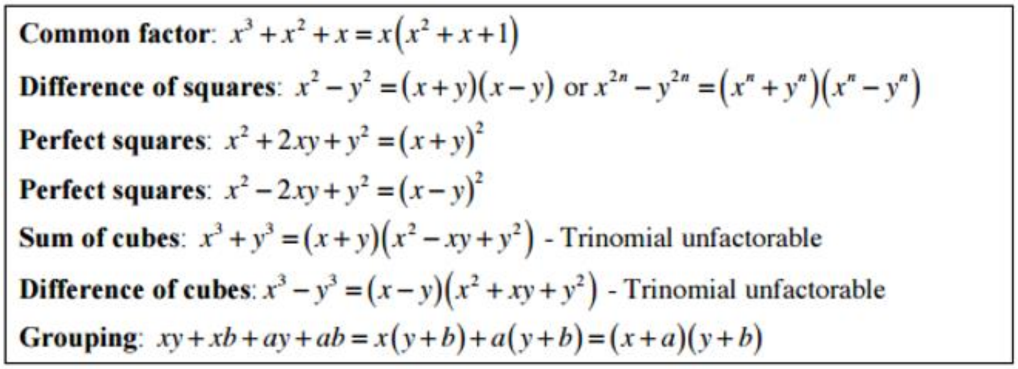
$\frac{1}{\left(27x^3\right)^{2/3}} = \frac{1}{\left(\sqrt[3]{27x^3}\right)^2} = \frac{1}{9x^2}$

6. $\left(16x^{-2}\right)^{3/4}$

$16^{3/4}x^{-6/4} = \left(\sqrt[4]{16}\right)^3 \frac{1}{x^{3/2}} = \frac{8}{x^3/2}$

Special Factorizations

The special forms that occur regularly are:



Common factor: $x^3 + x^2 + x = x\left(x^2 + x + 1\right)$

Difference of squares: $x^2 - y^2 = (x+y)(x-y)$ or $x^{2n} - y^{2n} = \left(x^n + y^n\right)\left(x^n - y^n\right)$

Perfect squares: $x^2 + 2xy + y^2 = (x+y)^2$

Perfect squares: $x^2 - 2xy + y^2 = (x-y)^2$

Sum of cubes: $x^3 + y^3 = (x+y)\left(x^2 - xy + y^2\right)$ - Trinomial unfactorable

Difference of cubes: $x^3 - y^3 = (x-y)\left(x^2 + xy + y^2\right)$ - Trinomial unfactorable

Grouping: $xy + xb + ay + ab = x(y+b) + a(y+b) = (x+a)(y+b)$

1. $8a^2 - 4a$

$4a(2a-1)$

2. $5x^4 - 5y^4$

$5\left(x^4 - y^4\right) = 5(x+y)(x-y)\left(x^2 + y^2\right)$

3. $x^3 - 125$

$(x-5)\left(x^2 + 5x + 25\right)$

4. $x^3 - x^2 + 3x - 3$

$x^2(x-1) + 3(x-1)$

$(x-1)\left(x^2 + 3\right)$

5. $x^3 + 5x^2 - 4x - 20$

$x^2(x+5) - 4(x+5)$

$(x+5)(x+2)(x-2)$

Section A: Simplify. Write answers with positive exponents only

1. $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

2. $\left(\dfrac{-4}{x^4}\right)^{-3}$

3. $\left(4a^{5/3}\right)^{3/2}$

4. $\left(8x^6\right)^{-4/3}$

5. $\dfrac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

Factor completely

6. x^3-25x

7. $3x^3-5x^2+2x$

8. $4x^4+7x^2-36$

9. $x^6-9x^4-81x^2+729$

10. $x^3-xy^2+x^2y-y^3$

Solve each equation

11. $\dfrac{6x-7}{4}+\dfrac{3x-5}{7}=\dfrac{5x+78}{28}$

12. $x^3-6x^2-27x=0$

13. $\sqrt{x+1}-3x=1$

14. $-\dfrac{2}{x^2}+\dfrac{1}{3(x-2)^2}=0$

15. Solve for y:
$xy+2x+1=y$

Section B: Complex Fractions

Example:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}=\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}}\cdot\frac{x+1}{x+1}=\frac{-7x-7-6}{5}=\frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3}{x-4}}{5-\frac{1}{x-4}}=\frac{\frac{-2}{x}+\frac{3}{x-4}}{5-\frac{1}{x-4}}\cdot\frac{x(x-4)}{x(x-4)}=\frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)}=\frac{-2x+8+3x^2}{5x^2-20x-x}=\frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following

1. $\frac{1}{x+h}-\frac{1}{x}$

2. $\dfrac{1-\frac{2x}{3x-4}}{x+\frac{32}{3x-4}}$

3. $\dfrac{\frac{1}{3+x}-\frac{1}{3}}{x}$

4. $\dfrac{\frac{25}{a}-a}{5+a}$

5. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

6. $\dfrac{2-\frac{4}{x+2}}{5+\frac{10}{x+2}}$

Section C: Functions

To evaluate a function for a given value, simply plug in the value for x.

Example: Given $g(x) = 3x^2 - x + 1$ find $g(-1)$

$$\begin{aligned} g(-1) &= 3(-1)^2 - (-1) + 1 \\ &= 3(1) + 1 + 1 \\ &= 3 + 1 + 1 \\ &= 5 \\ g(-1) &= 5 \end{aligned}$$

Recall: Composition of functions - $(f \circ g)(x) = f(g(x))$ read "***f*** of ***g*** of x" Means plug the inside function (in this case, *g(x)*) in for x in the outside function (in this case, *f*(*x*)).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Given $f(x) = x^2 - 2x + 5$, evaluate each of the following.

1. $f(-2) =$ 2. $f(x+2) =$ 3. $f(x+h) =$

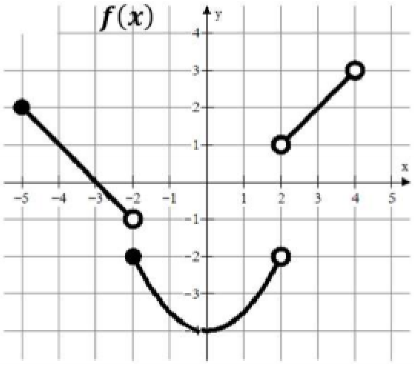
4. Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each.

a. $f(2) =$ b. $g(-3) =$ c. $f(x+1) =$

d. $f(g(x)) =$ e. $f(g(-2)) =$ f. $g(f(x)) =$

g. $f(x+h) - f(x) =$ h. $\dfrac{f(x+h) - f(x)}{h} =$ i. $f(g(x^3) =$

5. Use the graph of $f(x)$ to evaluate each of the following.



$f(0) =$ $f(4) =$

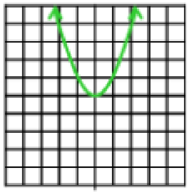
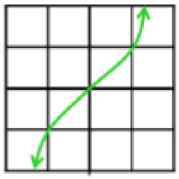
$f(-1) =$ $f(-2) =$

$f(2) =$ $f(3) =$

$f(x) = 2$ when $x = ?$ $f(x) = -3$ when $x = ?$

6. Find $\dfrac{f(x+h) - f(x)}{h}$ for the given function f.

i. $f(x) = 9x + 3$ j. $f(x) = 5 - 2x$

Quick Review		
Even Function	Symmetric about the y axis $f(-x) = f(x)$ for all x	Example: $y = x^2$ 
Odd Function	Symmetric about the origin (equivalent to a rotation of 180 degrees) $f(-x) = -f(x)$ for all x	Example: $y = x^3$ 

7. Determine if the function is even, odd or neither

a. $f(x) = \frac{x^2}{x^4+3}$ b. $f(x) = \frac{x}{x+1}$

c. $f(x) = 1 + 3x^2 + 3x^4$ d. $f(x) = 1 + 3x^3 + 3x^5$

Summer Assignment Part 2: Due July 27, 2023 Non Calculator – unless otherwise specified

Name: **Cell:** **email:**

Section A: Linear Functions

One of the most important concepts that you need from pre-calculus that is required for differential calculus is that of linear functions. The formulas you need to know backwards and forwards are:

Slope: Given two points (x_1, y_1) and (x_2, y_2), the line passing through the points can be written as: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Point Slope form: the equation of a line through a point (x_1, y_1) with slope *m* is given by $y - y_1 = m(x - x_1)$. This is the preferred form and far more useful form of the equation of a line in calculus.

Parallel Lines: Two distinct lines are parallel if and only if $m_1 = m_2$.

Normal Lines: Two lines are normal (perpendicular) if their slopes are opposite reciprocals. $m_1 \cdot m_2 = -1$

Horizontal Lines have a slope of zero, and form $y = c$.

Vertical Lines have undefined slope and form $x = c$.

Example

1. Find the equation of a line with the given slope through the given point.

a. $m = -4$, (1, 2)

$$\boxed{y - 2 = -4(x - 1)}$$

b. $m = 0$, $\left(-\frac{1}{2}, \frac{3}{4}\right)$

$$\boxed{y = \frac{3}{4}}$$

2. Write the equations of the line through the given point a. parallel and b. normal to the given line.

a. (4, 7) $4x - 2y = 1$

$m = 2$

$a) y - 7 = 2(x - 4)$

$b) y - 7 = -\frac{1}{2}(x - 4)$

b. $\left(\frac{2}{3}, 1\right)$ $x + 5y = 2$

$m = -\frac{1}{5}$

$a) y - 1 = -\frac{1}{5}\left(x - \frac{2}{3}\right)$

$b) y - 1 = 5\left(x - \frac{2}{3}\right)$

Section A: Linear Functions

1. Write the equation of the line in point slope form.

a. $(-3,6)$ and $(-1,2)$

b. $m=\frac{2}{3},\left(-6,\frac{1}{3}\right)$

2. Write the equations of the line through the given point a. parallel and b. normal to the given line.

a. $(-3,-4);\ y=-2$

b. $(-6,2);\ 5x+2y=7$

3. Find k if the lines $3x-5y=9$ and $2x+ky=11$ are a) parallel and b) perpendicular.

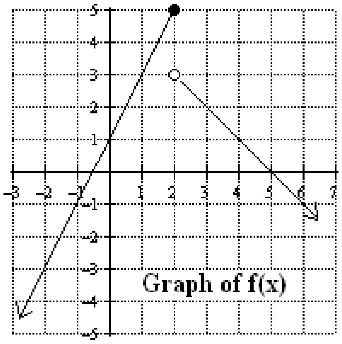
4. Find the equation of a line with an x-intercept of (2, 0) and a y-intercept of (0, 3).

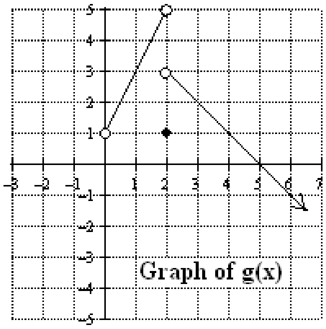
5. The former Republic Of Yugoslavia exported cars called Yugos to the United States between 1985 and 1989. The car is now a collector's item. The table below gives the quantity of Yugos sold and the price from 1985 to 1989. (**you may use a calculator**)

Year	Price in $	Number Sold
1985	3990	49000
1986	4110	43000
1987	4200	38500
1988	4330	32000

a. Using the table, explain why Q could be a function of p.
b. What does the rate of change of this function tell you about the sales of Yugos?

6.



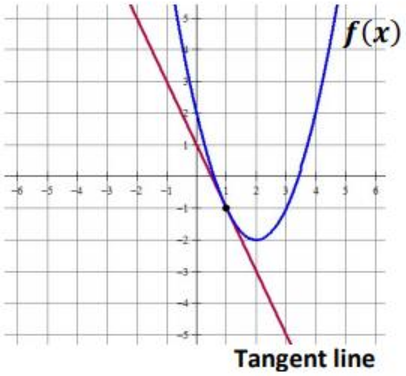


The graphs of two similar, yet different, piecewise defined functions, *f*(*x*) and *g*(*x*), are pictured above. Use the graphs to answer the following questions.

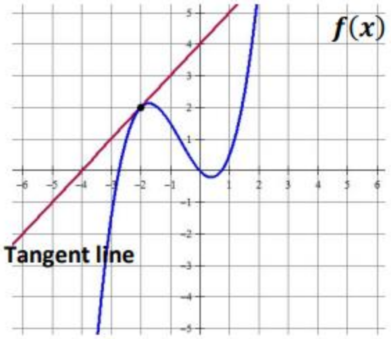
a. Find *f*(2). Justify your reasoning.
b. Identify the domain and range of *f*(*x*).
c. Find the equation of *f*(*x*) as a piecewise defined function.
d. Modify the equation of *f*(*x*) in part c) to obtain the equation for the graph of *g*(*x*).

7. Write the equation of the tangent line to the given function at the specified point.

a. The line tangent to $f(x)$ at $x = 1$

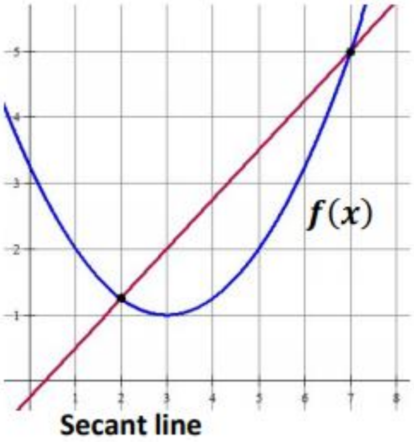


b. The line tangent to $f(x)$ at $x = -2$



8. Which choice represents the slope of the secant line shown?

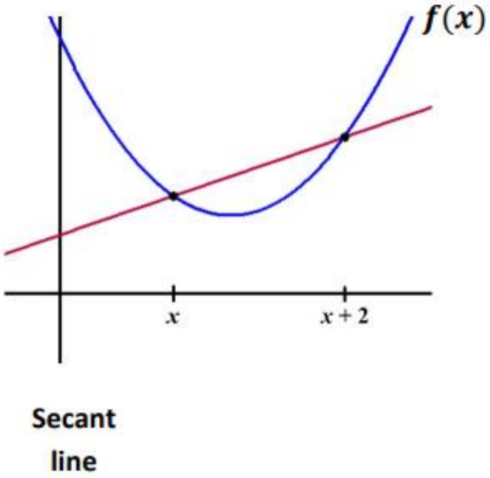
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



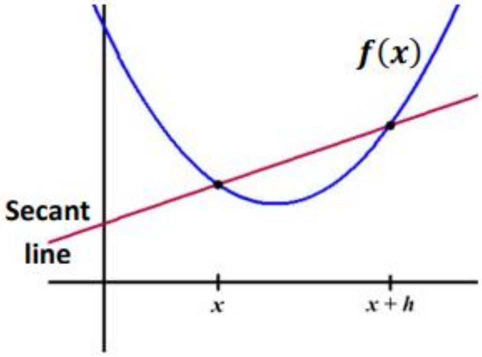
9. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



10. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

Exponential & Logarithmic Functions

Calculus spends a great deal of time on exponential functions in the form of b^x. Don't expect that when you start working with them in calculus, your teacher will review them. So learn them now! Students must know that the definition of a **logarithm** is based on exponential equations. If $y = b^x$ then $x = \log_b y$. So when you are trying to find the value of $\log_2 32$, state that $\log_2 32 = x$ and $2^x = 32$ and therefore $x = 5$.

If the base of a log statement is not specified, it is defined to be 10. When we asked for log 100, we are solving the equation: $10^x = 100$ and $x = 2$. The function $y = \log x$ has domain $(0, \infty)$ and range $(-\infty, \infty)$. In calculus, we primarily use logs with base *e*, which are called natural logs (ln). So finding ln 5 is the same as solving the equation $e^x = 5$. Students should know that the value of $e = 2.71828\ldots$

There are three rules that students must keep in mind that will simplify problems involving logs and natural logs. These rules work with logs of any base including natural logs.

i. $\log a + \log b = \log(a \cdot b)$ ii. $\log a - \log b = \log\left(\frac{a}{b}\right)$ iii. $\log a^b = b \log a$

1. Find a. $\log_4 8$

$$\boxed{\begin{aligned}&\log_4 8 = x\\&4^x = 8 \Rightarrow 2^{2x} = 2^3\\&x = \frac{3}{2}\end{aligned}}$$

b. $\ln\sqrt{e}$

$$\boxed{\begin{aligned}&\ln\sqrt{e} = x\\&e^x = e^{1/2}\\&x = \frac{1}{2}\end{aligned}}$$

c. $10^{\log 4}$

$$\boxed{\begin{aligned}&\log 4 = x\\&10^x = 4 \text{ so } 10^{\log 4} = 4\\&\text{10 to a power and log are inverses}\end{aligned}}$$

d. $\log 2 + \log 50$

$$\boxed{\begin{aligned}&\log(2 \cdot 50) = \log 100\\&2\end{aligned}}$$

e. $\log_4 192 - \log_4 3$

$$\boxed{\begin{aligned}&\log_4\left(\frac{192}{3}\right)\\&\log_4 64 = 3\end{aligned}}$$

f. $\ln\sqrt[5]{e^3}$

$$\boxed{\ln e^{3/5} = \frac{3}{5}\ln e = \frac{3}{5}}$$

2. Solve a. $\log_9\left(x^2 - x + 3\right) = \frac{1}{2}$

$$\boxed{\begin{aligned}&x^2 - x + 3 = 9^{1/2}\\&x(x-1) = 0\\&x = 0, x = 1\end{aligned}}$$

b. $\log_{36} x + \log_{36}(x-1) = \frac{1}{2}$

$$\boxed{\begin{aligned}&\log_{36} x(x-1) = \frac{1}{2}\\&x(x-1) = 36^{1/2} = 6\\&x^2 - x - 6 = 0\\&(x-3)(x+2) = 0\\&\text{Only } x = 3 \text{ is in the domain}\end{aligned}}$$

c. $\ln x - \ln(x-1) = 1$

$$\boxed{\begin{aligned}&\ln\left(\frac{x}{x-1}\right) = 1\\&\frac{x}{x-1} = e \Rightarrow x = ex - e\\&x = \frac{e}{e-1}\end{aligned}}$$

d. $5^x = 20$

$$\boxed{\begin{aligned}&\log\left(5^x\right) = \log 20\\&x\log 5 = \log 20\\&x = \frac{\log 20}{\log 5} \text{ or } x = \frac{\ln 20}{\ln 5}\end{aligned}}$$

e. $e^{-2x} = 5$

$$\boxed{\begin{aligned}&\ln e^{-2x} = \ln 5\\&-2x = \ln 5 \Rightarrow x = \frac{-\ln 5}{2}\end{aligned}}$$

f. $2^x = 3^{x-1}$

$$\boxed{\begin{aligned}&\log\left(2^x\right) = \log\left(3^{x-1}\right)\\&x\log 2 = (x-1)\log 3\\&x\log 2 = x\log 3 - \log 3 \Rightarrow x = \frac{\log 3}{\log 3 - \log 2}\end{aligned}}$$

Section B: Exponential & Logarithmic Functions

1. Evaluate each expression

a. $5^{\log_5 40}$

b. $\log_8 4$

c. $\ln \frac{1}{\sqrt[3]{e^2}}$

d. $\log_{12} 2 + \log_{12} 9 + \log_{12} 8$

e. $\log_2 \frac{2}{3} + \log_2 \frac{3}{32}$

2. Solve

a. $\log_5 (3x - 8) = 2$

b. $\log_2 (x-1) + \log_2 (x+3) = 5$

c. $\ln x^3 - \ln x^2 = \frac{1}{2}$

d. $3^{x-2} = 18$

e. $e^{3x+1} = 10$

f. $8^x = 5^{2x-1}$

g. $2(3^{4x-5}) + 4 = 11$

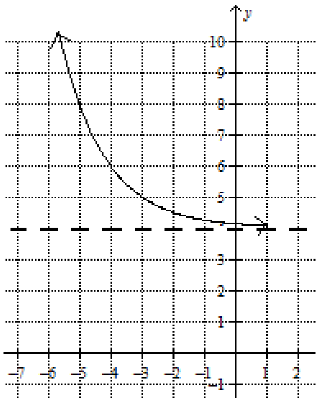
h. $e^{\ln(2x+1)} = 5x$

3. A 2006 Lexus costs \$61,055 and the car depreciates a total of 46% during its first 7 years. (**you may use a calculator**)

a. Suppose the depreciation is exponential. Find a formula for the value f the car at time t.
b. Suppose instead that the depreciation is linear. Find a formula for the value f the car at time t.
c. If this were your car and you were trading it in after 4 years, which depreciation model would you prefer (exponential or linear)

4. Pictured to the right is the graph of an exponential function, $f(x) = a(b)^x + c$. Use the graph to answer the following questions.

a. If $a > 0$, describe the value of b. Give a reason for your answer.
b. What is the value of c? Give a reason for your answer.
c. The inverse function of f is a logarithmic function. What would the argument of the function $f^{-1}(x)$ be? Give a reason for your answer.
d. Identify the domain and range of the inverse function, $f^{-1}(x)$.



Summer Assignment Part 3: Due August 10, 2023

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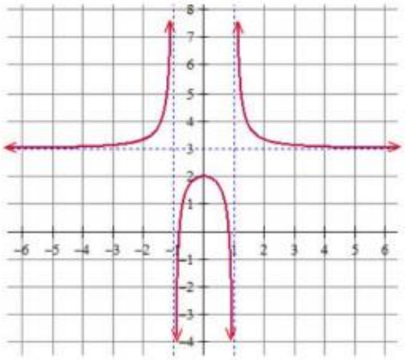
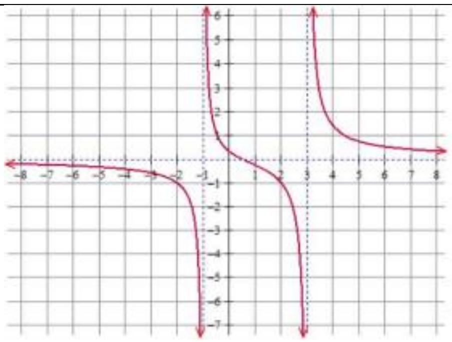
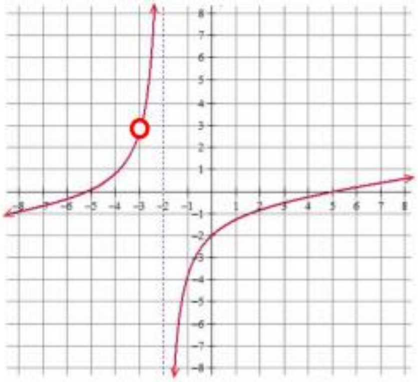
1. Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

		
Domain:	Domain:	Domain:
Range:	Range:	Range:
Horizontal Asymptote(s):	Horizontal Asymptote(s):	Horizontal Asymptote(s):
Vertical Asymptotes(s):	Vertical Asymptotes(s):	Vertical Asymptotes(s):

2. For each function, determine its domain and range.

Function	Domain	Range
$y = \sqrt{x-4}$		
$y = (x-3)^2$		
$y = \ln x$		
$y = e^x$		
$y = \sqrt{4-x^2}$		

3. Operations with Functions (again ☺)

a. Given $f(x)=|x-3|-5$ find $f(1)-f(5)$

b. Given $f(x)=x^2-3x+4$ find $f(x+2)\cdot f(2)$

c. Find $f(x+\Delta x)$ for $f(x)=x^2-2x-3$

d. Find $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ for $f(x)=8x^2+1$

4. Find $\dfrac{f(x+h)-f(x)}{h}$ for the given function f .

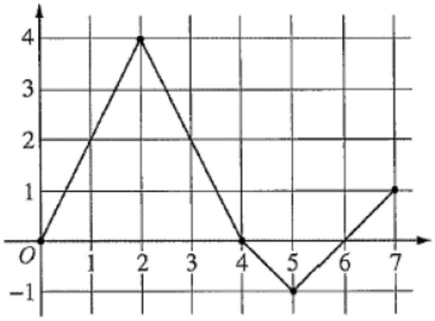
a. $f(x)=9x+3$ b. $f(x)=5-2x$ c. $f(x)=\dfrac{1}{x}$ d. $f(x)=\sqrt{x+3}$

5. If $f(x)=\{(3,5),(2,4),(1,7)\}$ $g(x)=\sqrt{x-3}$ $h(x)=\{(3,2),(4,3),(1,6)\}$ $k(x)=x^2+5$, determine each of the following.

a. $(f+h)(1)$ b. $f(h(3))$

c. $g(f(2))$ d. $k(g(x))$

Open Ended



Graph of f

6. Let f be a function defined on the closed interval [0, 7]. The graph of f, consisting of four line segments, is shown above.

a. Write the piecewise function for $f(x)$.
b. Find the values of x for which $f(x) = 1$.
c. For what interval(s) of x does f have a rate of change of 2? Explain your answer.
d. On which interval of x is the rate of change of f the greatest? Justify your answer.
e. Sketch a graph of the function f if it is odd.
f. Sketch a graph of the function f if it is even.

7. The function g is a transformation of the above function f. Write an equation for g in terms of f.

a. The maximum value of g is located at the point (2,5).
b. The maximum value of g is located at the point (2,2).
c. The minimum value of g is located at the point (2, -4).
d. The range of g is [0,4].

Summer Assignment Part 4: Due September 6, 2023

Name: **Cell:** **email**

1. Without a calculator, determine the exact value of each expression. Please note: You MUST be able to do this in your sleep by the time you get back to school WITHOUT A CALCULATOR!! This is one of the single most important sills necessary for you to be successful in AP Calculus!

a. $\sin 0$

b. $\cos\frac{7\pi}{6}$

c. $\tan\frac{\pi}{6}$

d. $\cos\left(\sin^{-1}\frac{1}{2}\right)$

e. $\sin\frac{\pi}{2}$

f. $\cos\frac{\pi}{3}$

g. $\tan\frac{2\pi}{3}$

h. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

i. $\sin\frac{3\pi}{4}$

j. $\tan\frac{7\pi}{4}$

k. $\tan\frac{\pi}{2}$

l. $\sin(11\pi)$

m. $\cos\pi$

n. $\cos\frac{21\pi}{4}$

o. $\tan\left(\frac{13\pi}{3}\right)$

2. If $\sin x = -\frac{2}{5}$ and $\pi < x < \frac{3\pi}{2}$, find the values of the other five trigonometric functions.

3. Evaluate each of the following.

a. $\cos\left(\csc^{-1}\left(\frac{7}{4}\right)\right)$

b. $\tan\left(\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

c. $\cos^{-1}\left(\cot\left(\frac{3\pi}{4}\right)\right)$

d. $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

4. If $\csc\theta = \frac{13}{5}$ and θ is in the second quadrant, find the remaining trigonometric ratios.

5. If $\cot\theta = \frac{4}{3}$ and θ is in the third quadrant, find the remaining trigonometric ratios.

6. Solve and give answers on the interval $[0,2\pi)$.

a) $\sin x = -\frac{1}{2}$

b) $2\cos x = \sqrt{3}$

c) $\cos 2x = \frac{1}{\sqrt{2}}$

d) $\sin^2 x = \frac{1}{2}$

e) $4\cos^2 x - 3 = 0$

f) $2\cos^2 x - 1 - \cos x = 0$

g) $\sin^2 x + \cos 2x - \cos x = 0$

h) $\cos^2 x - \cos 2x + \sin x = -\sin^2 x$

7. Simplify the following trigonometric expressions/identities.

a) $\sin^2 x + \cos^2 x =$

b) $\frac{1}{\sin x} =$

c) $\sin^4 x - \cos^4 x =$

d) $\tan^2 x + 1 =$

e) $\sin 2x =$

f) $\cos(x + \Delta x) =$

g) $\tan^4 x + 2\tan^2 x + 1$

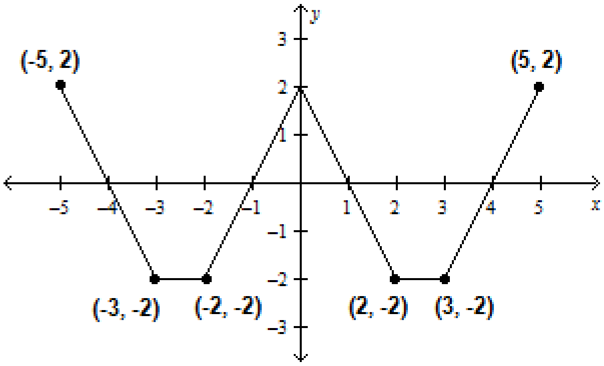
h) $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x}$

8. Prove the identity.

a) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

b) $\frac{1}{\cos^2 x - \cos 2x} = \csc^2 x$

9.



The graph of a periodic function, $f(x)$, on the interval [–5, 5] is pictured above. Answer the following questions using the graph.

a. Does $f^{-1}(x)$ exist? Give both a numerical and graphical justification for your answer.
b. Identify the period, amplitude, maximum value, and minimum value of $f(x)$.
c. What is the value of $f(42)$? Explain your reasoning.
d. Express $f(x)$ on the interval [0, 5] as a three part piecewise defined function.

Unit Circle Trigonometry. You will be expected to know the values of all 6 trigonometric functions at radian measures 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, and π, and all radian measures that are multiples of these -- and you must know these without a calculator. These can be done by referencing the unit circle, a circle of radius 1 centered at the origin. Every point (x, y) on that circle can be interpreted as $(\cos\theta, \sin\theta)$, where θ is the angle formed with the positive *x*-axis. It's only necessary to memorize the important first-quadrant points -- the rest can be reckoned by reflecting appropriately. (See diagram below.) The remaining trigonometric functions can be computed from sines and cosines.